

Exercise 29

Explain, in terms of linear approximations or differentials, why the approximation is reasonable.

$$\sec 0.08 \approx 1$$

Solution

Compute the derivative of $y = \sec x$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(\sec x) \\ &= \sec x \tan x\end{aligned}$$

Consequently, the differential of $y = \sec x$ is

$$dy = \sec x \tan x \, dx.$$

In order to estimate $\sec 0.08$, set $x = 0$ and $dx = 0.08$.

$$dy = \sec 0 \tan 0 (0.08) = 0$$

Note that dy here is the vertical distance from the function's actual value at $x = 0$ to the linear approximation's value at $x = 0.08$.

$$\sec 0.08 \approx \sec 0 + 0 = 1$$